

POSSIBLE TETRAQUARK STATES IN THE $\pi^+\chi_{c1}$ INVARIANT MASS DISTRIBUTION

Zhi-Gang Wang ¹

Department of Physics, North China Electric Power University, Baoding 071003,
P. R. China ²

Kavli Institute for Theoretical Physics China, Institute of Theoretical Physics,
Chinese Academy of sciences, Beijing 100080, P. R. China

Abstract

In this article, we assume that there exist hidden charmed tetraquark states with the spin-parity $J^P = 1^-$, and calculate their masses with the QCD sum rules. The numerical result indicates that the masses of the vector hidden charmed tetraquark states are about $M_Z = (5.12 \pm 0.15) \text{ GeV}$ or $M_Z = (5.16 \pm 0.16) \text{ GeV}$, which are inconsistent with the experimental data on the $\pi^+\chi_{c1}$ invariant mass distribution. The hidden charmed mesons Z_1 , Z_2 or Z may be scalar hidden charmed tetraquark states, hadro-charmonium resonances or molecular states.

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1 Introduction

The Babar, Belle, CLEO, D0, CDF and FOCUS collaborations have discovered (or confirmed) a large number of charmonium-like states [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11], and revitalized the interest in the spectroscopy of the charmonium states [12, 13, 14, 15].

The $X(3940)$ decaying into $D^*\bar{D}$ and the $Z(3930)$ decaying into $D\bar{D}$ have been tentatively identified as candidates for the missing charmonium states η_c'' and χ_{c2}' respectively. The $X(3872)$ decaying into $\pi^+\pi^-J/\psi$, $\pi^+\pi^-\pi^0J/\psi$, the $Y(4260)$, $Y(4008)$ decaying into $\pi^+\pi^-J/\psi$, the $Y(3940)$ decaying into $\omega J/\psi$, the $Y(4325)$, $Y(4360)$, $Y(4660)$ decaying into $\pi^+\pi^-\psi'$ have odd properties comparing with the expectations of the charmonium models [12, 13, 14, 15].

Many possible assignments for those states have been suggested, such as multi-quark states (whether the molecular type [16, 17] or the diquark-antidiquark type [18, 19, 20]), hybrid states [21, 22, 23], charmonium states modified by nearby thresholds [24, 25], threshold cusps [26], etc.

The observed decay channels are $J/\psi\pi^+\pi^-$ or $\psi'\pi^+\pi^-$, an essential ingredient for understanding the structures of those mesons is whether or not the $\pi\pi$ comes from a resonance state. For example, there is an indication that the $Y(4660)$ has a well

¹E-mail, wangzgyiti@yahoo.com.cn.

²Mailing address.

defined intermediate state in the $\pi\pi$ invariant mass distribution, which is consistent with the scalar meson $f_0(980)$ [27], the $Y(4660)$ can be taken as a $f_0(980)\psi'$ bound state [28]; though other interpretations such as a baryonium state [29] or a canonical 5^3S_1 $c\bar{c}$ state [30] are not excluded.

The $Z^+(4430)$ observed in the $\psi'\pi^+$ decay mode is the most interesting subject [31]. It can't be a pure $c\bar{c}$ state due to the positive charge. There are many theoretical interpretations for its structures, such as the hadro-charmonium resonance [14, 32], the molecular $D^*D_1(D'_1)$ state [33, 34, 35, 36, 37, 38], the tetraquark state [39, 40, 41, 42, 43], the cusp in the D^*D_1 channel [44], the radially excited state of the D_s [45], etc. We can distinguish the multi-quark states from the hybrids or charmonia with the criterion of non-zero charge.

Recently the Belle collaboration reported the first observation of two resonance-like structures (thereafter we will denote them as Z_1 and Z_2 respectively) in the $\pi^+\chi_{c1}$ invariant mass distribution near 4.1 GeV in the exclusive $\bar{B}^0 \rightarrow K^-\pi^+\chi_{c1}$ decays [46]. Their quark contents must be some special combinations of the $c\bar{c}u\bar{d}$, just like the $Z^+(4430)$, they cannot be the conventional mesons. The Breit-Wigner masses and widths are about $M_1 = 4051 \pm 14_{-41}^{+20}$ MeV, $\Gamma_1 = 82_{-17}^{+21+47}$ MeV, $M_2 = 4248_{-29-35}^{+44+180}$ MeV and $\Gamma_2 = 177_{-39-61}^{+54+316}$ MeV. The significance of each of the $\pi^+\chi_{c1}$ structures exceeds 5σ , including the effects of systematics from various fit models.

The Z (denote the Z_1 and Z_2) lie about (0.5–0.6) GeV above the $\pi^+\chi_{c1}$ threshold, the decay $Z \rightarrow \pi^+\chi_{c1}$ can take place with the "fall-apart" mechanism and it is OZI super-allowed, which can take into account the large width naturally. The spins of the Z_1 and Z_2 are not determined yet, they can be scalar or vector states [46].

If they are scalar mesons, the decays $Z \rightarrow \pi^+\chi_{c1}$ occur through the relative P -wave with the phenomenological lagrangian $\mathcal{L} = g\chi^\alpha(\pi\partial_\alpha Z - Z\partial_\alpha\pi)$. On the other hand, if they are vector mesons, the decays occur through the relative S -wave with the phenomenological lagrangian $\mathcal{L} = g\chi^\alpha Z_\alpha\pi$.

The typical decay mode $Z \rightarrow D^+\bar{D}^0$ is kinematically allowed, and the width may be comparable with the corresponding ones of the decay mode $Z \rightarrow \pi^+\chi_{c1}$, we can determine the spins of the Z with the angular distributions of the final states $D^+\bar{D}^0$. If the decays $Z \rightarrow D^+\bar{D}^0$ are not observed (or the widths are rather narrow), the Z may be hadro-charmonium resonances [14], i.e. bound states of a relatively compact charmonium (χ_{c1}) inside a light hadron (π^+) having a larger spatial size, the decays $Z \rightarrow \pi^+\chi_{c1}$ occur with the "fall-apart" mechanism; the decays $Z \rightarrow D^+\bar{D}^0$ take place through the final-state re-scattering effects, $Z \rightarrow \pi^+\chi_{c1} \rightarrow D^+\bar{D}^0$, the widths may be very narrow.

The masses of the D^+ and \bar{D}^0 are about $M_D = 1.87$ GeV, the Z may also be P -wave $D^+\bar{D}^0$ molecular states with the spin-parity 1^- , as the additional contribution from the relative P -wave is about 0.5 GeV in the potential quark models, the decays $Z \rightarrow D^+\bar{D}^0$ occur through "fall-apart" mechanism and have much larger widths than the corresponding decays $Z \rightarrow D^+\bar{D}^0 \rightarrow \pi^+\chi_{c1}$, which occur through final-state re-scattering effects. One may also think that they are D^*D_1 (or $D^*D'_1$) molecular states, $M_{D^*} = 2.01$ GeV, $M_{D_1} = 2.42$ GeV, $M_{D'_1} = 2.43$ GeV, the bound energy

is about $(0.2 - 0.4)$ GeV, which may be beyond capacity of the one- π and one- σ exchange.

The mass is a fundamental parameter in describing a hadron, in order to identify the Z_1 and Z_2 as tetraquark states, we must prove that the masses of the corresponding tetraquark states lie in the region $(4.1 - 4.3)$ GeV. Furthermore, whether or not there exist such hidden tetraquark configurations is of great importance itself, because it provides a new opportunity for a deeper understanding of low energy QCD.

In this article, we assume that the hidden charmed mesons Z_1 and Z_2 are vector tetraquark states, which consist of a pseudoscalar (scalar) diquark and an axial-vector (vector) antidiquark, and study their masses with the QCD sum rules [47, 48]. As their spins are not determined yet, they may also be scalar hidden charmed tetraquark states, we study this possibility with the QCD sum rules [49].

In the QCD sum rules, operator product expansion is used to expand the time-ordered currents into a series of quark and gluon condensates which parameterize the long distance properties of the QCD vacuum. Based on current-hadron duality, we can obtain copious information about the hadronic parameters at the phenomenological side.

It is difficult to distinguish the mesons Z_1 and Z_2 with the QCD sum rules approach, as the mass gap between them is very small. The mesons Z_1 and Z_2 lie in the region $(4.0 - 4.3)$ GeV, we study whether or not there exist 1^- hidden charmed tetraquark states Z in this energy region and possible tetraquark identification of the mesons Z_1 and Z_2 . Furthermore, the $\pi^+\chi_{c1}$ invariant mass distribution amplitude can also be represented by a single Breit-Wigner mass formula, $M_Z = 4150^{+31}_{-16}$ MeV and $\Gamma_Z = 352^{+99}_{-43}$ MeV [46].

The article is arranged as follows: we derive the QCD sum rules for the masses and pole residues of the Z in section 2; in section 3, numerical results and discussions; section 4 is reserved for conclusion.

2 QCD sum rules for the tetraquark states Z

In the following, we write down the two-point correlation functions $\Pi_{\mu\nu}(p)$ (denote $\Pi_{\mu\nu}^J(p)$ and $\Pi_{\mu\nu}^\eta(p)$) in the QCD sum rules,

$$\Pi_{\mu\nu}^{J/\eta}(p) = i \int d^4x e^{ip \cdot x} \langle 0 | T \{ J/\eta_\mu(x) J/\eta_\nu^\dagger(0) \} | 0 \rangle, \quad (1)$$

$$J_\mu(x) = \epsilon^{ijk} \epsilon^{imn} u_j^T(x) C c_k(x) \bar{c}_m(x) \gamma_\mu C \bar{d}_n^T(x), \quad (2)$$

$$\eta_\mu(x) = \epsilon^{ijk} \epsilon^{imn} u_j^T(x) C \gamma_5 c_k(x) \bar{c}_m(x) \gamma_5 \gamma_\mu C \bar{d}_n^T(x), \quad (3)$$

$$f_Z M_Z^4 \epsilon_\mu = \langle 0 | J/\eta_\mu(0) | Z(p) \rangle, \quad (4)$$

we choose the vector currents $J_\mu(x)$ ($C - C\gamma_\mu$ type) and $\eta_\mu(x)$ ($C\gamma_5 - C\gamma_\mu\gamma_5$ type) to interpolate the tetraquark states Z , the f_Z is the pole residue and the ϵ_μ is the polarization vector. If there exist vector tetraquark states Z in the $\pi^+\chi_{c1}$ invariant

mass distribution, it is convenient to construct the tetraquark currents with the pseudoscalar (scalar) diquark and axial-vector (vector) antidiquark, as the mesons π and χ_{c1} have the spin-parity 0^- and 1^+ respectively, the decays $Z \rightarrow \pi^+ \chi_{c1}$ can occur through relative S -wave. We can also interpolate the vector tetraquark states with the currents $J_\mu^1(x)$ and $\eta_\mu^1(x)$,

$$J_\mu^1(x) = \epsilon^{ijk} \epsilon^{imn} u_j^T(x) C \gamma_\mu c_k(x) \bar{c}_m(x) C \bar{d}_n^T(x), \quad (5)$$

$$\eta_\mu^1(x) = \epsilon^{ijk} \epsilon^{imn} u_j^T(x) C \gamma_\mu \gamma_5 c_k(x) \bar{c}_m(x) \gamma_5 C \bar{d}_n^T(x), \quad (6)$$

which consist of a pseudoscalar (scalar) antidiquark and an axial-vector (vector) diquark. Our analytical results indicate that the current $J_\mu(x)$ ($\eta_\mu(x)$) and $J_\mu^1(x)$ ($\eta_\mu^1(x)$) lead to the same expression. The special superpositions $tJ_\mu(x) + (1-t)J_\mu^1(x)$ and $t\eta_\mu(x) + (1-t)\eta_\mu^1(x)$ cannot improve the predictions remarkably, where $t = 0-1$.

The correlation functions $\Pi_{\mu\nu}(p)$ can be decomposed as follows,

$$\Pi_{\mu\nu}(p) = (-g_{\mu\nu} + \frac{p_\mu p_\nu}{p^2}) \Pi_1(p^2) + \frac{p_\mu p_\nu}{p^2} \Pi_0(p^2) + \dots, \quad (7)$$

due to Lorentz covariance. The invariant functions Π_1 and Π_0 stand for the contributions from the vector and scalar mesons, respectively. In this article, we choose the tensor structure $g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}$ to study the masses of the vector mesons.

Basing on the quark-hadron duality [47, 48], we can insert a complete series of intermediate states with the same quantum numbers as the current operators $J_\mu(x)$ and $\eta_\mu(x)$ into the correlation functions $\Pi_{\mu\nu}(p)$ to obtain the hadronic representation. After isolating the ground state contribution from the pole terms of the Z , we get the following result,

$$\Pi_{\mu\nu}(p) = \frac{f_Z^2 M_Z^8}{M_Z^2 - p^2} \left[-g_{\mu\nu} + \frac{p_\mu p_\nu}{p^2} \right] + \dots. \quad (8)$$

In the following, we briefly outline operator product expansion for the correlation functions $\Pi_{\mu\nu}(p)$ in perturbative QCD theory. The calculations are performed at large space-like momentum region $p^2 \ll 0$. We write down the "full" propagators $S_{ij}(x)$ and $C_{ij}(x)$ of a massive quark in the presence of the vacuum condensates firstly [48],

$$\begin{aligned} S_{ij}(x) = & \frac{i\delta_{ij} \not{x}}{2\pi^2 x^4} - \frac{\delta_{ij} m_q}{4\pi^2 x^2} - \frac{\delta_{ij}}{12} \langle \bar{q}q \rangle + \frac{i\delta_{ij}}{48} m_q \langle \bar{q}q \rangle \not{x} - \\ & \frac{\delta_{ij} x^2}{192} \langle \bar{q}g_s \sigma Gq \rangle + \frac{i\delta_{ij} x^2}{1152} m_q \langle \bar{q}g_s \sigma Gq \rangle \not{x} - \\ & \frac{i}{32\pi^2 x^2} G_{\mu\nu}^{ij} (\not{x} \sigma^{\mu\nu} + \sigma^{\mu\nu} \not{x}) + \dots, \end{aligned} \quad (9)$$

$$\begin{aligned} C_{ij}(x) = & \frac{i}{(2\pi)^4} \int d^4k e^{-ik \cdot x} \left\{ \frac{\delta_{ij}}{\not{k} - m_c} - \frac{g_s G_{ij}^{\alpha\beta}}{4} \frac{\sigma_{\alpha\beta} (\not{k} + m_c) + (\not{k} + m_c) \sigma_{\alpha\beta}}{(k^2 - m_c^2)^2} \right. \\ & \left. + \frac{\pi^2}{3} \langle \frac{\alpha_s GG}{\pi} \rangle \delta_{ij} m_c \frac{k^2 + m_c \not{k}}{(k^2 - m_c^2)^4} + \dots \right\}, \end{aligned} \quad (10)$$

where $\langle \bar{s}g_s\sigma Gs \rangle = \langle \bar{s}g_s\sigma_{\alpha\beta}G^{\alpha\beta}s \rangle$ and $\langle \frac{\alpha_s GG}{\pi} \rangle = \langle \frac{\alpha_s G_{\alpha\beta}G^{\alpha\beta}}{\pi} \rangle$, then contract the quark fields in the correlation functions $\Pi_{\mu\nu}(p)$ with Wick theorem, and obtain the result:

$$\begin{aligned} \Pi_{\mu\nu}^J(p) &= i\epsilon^{ijk}\epsilon^{imn}\epsilon^{i'j'k'}\epsilon^{i'm'n'} \int d^4x e^{ip\cdot x} \text{Tr} [CS_{kk'}^T(x)CC_{jj'}(x)] \\ &\quad \text{Tr} [\gamma_\mu CS_{n'n}^T(-x)C\gamma_\nu C_{m'm}(-x)] , \end{aligned} \quad (11)$$

$$\begin{aligned} \Pi_{\mu\nu}^\eta(p) &= i\epsilon^{ijk}\epsilon^{imn}\epsilon^{i'j'k'}\epsilon^{i'm'n'} \int d^4x e^{ip\cdot x} \text{Tr} [\gamma_5 CS_{kk'}^T(x)C\gamma_5 C_{jj'}(x)] \\ &\quad \text{Tr} [\gamma_5 \gamma_\mu CS_{n'n}^T(-x)C\gamma_\nu \gamma_5 C_{m'm}(-x)] . \end{aligned} \quad (12)$$

Substitute the full u , d and c quark propagators into the correlation functions $\Pi_{\mu\nu}(p)$ and complete the integral in the coordinate space, then integrate over the variables in the momentum space, we can obtain the correlation functions $\Pi_1(p^2)$ at the level of quark-gluon degrees of freedom. Once analytical results are obtained, then we can take current-hadron duality below the threshold s_0 and perform Borel transform with respect to the variable $P^2 = -p^2$, finally we obtain the following sum rules:

$$f_Z^2 M_Z^8 e^{-\frac{M_Z^2}{M^2}} = \int_{4m_c^2}^{s_0} ds \rho_{J/\eta}(s) e^{-\frac{s}{M^2}} , \quad (13)$$

$$\begin{aligned} \rho_{J/\eta}(s) &= \frac{1}{3072\pi^6} \int_{\alpha_{min}}^{\alpha_{max}} d\alpha \int_{\beta_{min}}^{1-\alpha} d\beta \alpha\beta(1-\alpha-\beta)^3 (s - \tilde{m}_c^2)^2 (7s - \tilde{m}_c^2) (5s - 3\tilde{m}_c^2) \\ &\quad \mp \frac{m_c \langle \bar{q}q \rangle}{32\pi^4} \int_{\alpha_{min}}^{\alpha_{max}} d\alpha \int_{\beta_{min}}^{1-\alpha} d\beta (1-\alpha-\beta)(s - \tilde{m}_c^2) \\ &\quad [s(4\beta - 3\alpha) + \tilde{m}_c^2(\alpha - 2\beta)] \\ &\quad \mp \frac{m_c \langle \bar{q}g_s\sigma Gq \rangle}{64\pi^4} \int_{\alpha_{min}}^{\alpha_{max}} d\alpha \int_{\beta_{min}}^{1-\alpha} d\beta [s(2\alpha - 3\beta) - \tilde{m}_c^2(\alpha - 2\beta)] \\ &\quad - \frac{m_c^2 \langle \bar{q}q \rangle^2}{12\pi^2} \int_{\alpha_{min}}^{\alpha_{max}} d\alpha , \end{aligned} \quad (14)$$

where $\alpha_{max} = \frac{1+\sqrt{1-\frac{4m_c^2}{s}}}{2}$, $\alpha_{min} = \frac{1-\sqrt{1-\frac{4m_c^2}{s}}}{2}$, $\beta_{min} = \frac{\alpha m_c^2}{\alpha s - m_c^2}$ and $\tilde{m}_c^2 = \frac{(\alpha+\beta)m_c^2}{\alpha\beta}$.

We carry out operator product expansion to the vacuum condensates adding up to dimension-6. In calculation, we take assumption of vacuum saturation for high dimension vacuum condensates, they are always factorized to lower condensates with vacuum saturation in the QCD sum rules, factorization works well in large N_c limit. In this article, we take into account the contributions from the quark condensates, mixed condensates, and neglect the contributions from the gluon condensate. In calculation, we observe the contributions from the gluon condensate are suppressed by large denominators and would not play any significant roles [50, 51, 52, 53, 54]. Furthermore, we neglect the terms proportional to the m_q ($= m_u = m_d$), their contributions are of minor importance and can be neglected safely.

Differentiating the Eq.(13) with respect to $\frac{1}{M^2}$, then eliminate the pole residue f_Z , we can obtain two sum rules for the masses of the Z ,

$$M_Z^2 = \frac{\int_{4m_c^2}^{s_0} ds \rho_{J/\eta}(s) s e^{-\frac{s}{M^2}}}{\int_{4m_c^2}^{s_0} ds \rho_{J/\eta}(s) e^{-\frac{s}{M^2}}} . \quad (15)$$

3 Numerical results and discussions

The c -quark mass appearing in the perturbative terms (see e.g. Eq.(14)) is usually taken to be the pole mass in the QCD sum rules, while the choice of the m_c in the leading-order coefficients of the higher-dimensional terms is arbitrary [55]. It is convenient to take the pole mass $m_c = (1.3 \pm 0.1) \text{ GeV}$ [55]. The \overline{MS} mass $m_c(m_c^2)$ relates with the pole mass \hat{m} through the relation $m_c(m_c^2) = \hat{m} \left[1 + \frac{C_F \alpha_s(m_c^2)}{\pi} + (K - 2C_F) \left(\frac{\alpha_s}{\pi} \right)^2 + \dots \right]^{-1}$, where K depends on the flavor number n_f . In this article, we take the approximation $m_c \approx \hat{m}$ without the α_s corrections for consistency. The value listed in the PDG is $m_c(m_c^2) = 1.27_{-0.11}^{+0.07} \text{ GeV}$ [57], it is reasonable to take the value $m_c = (1.3 \pm 0.1) \text{ GeV}$ in Ref.[55], we also present the result with larger uncertainty. The vacuum condensates are scale dependent, the average virtuality of the quarks is characterized by the Borel parameter M^2 , it makes sense to choose $\mu^2 = \mathcal{O}(M^2)$. In this article, the energy scale is taken as $\mu = 2 \text{ GeV}$, $\langle \bar{q}q \rangle = -(0.26 \pm 0.01 \text{ GeV})^3$, $\langle \bar{q}g_s \sigma G q \rangle = m_0^2 \langle \bar{q}q \rangle$, $m_0^2 = (0.8 \pm 0.2) \text{ GeV}^2$ [47, 48, 56].

In the conventional QCD sum rules [47, 48], there are two criteria (pole dominance and convergence of the operator product expansion) for choosing the Borel parameter M^2 and threshold parameter s_0 . In calculation, we usually consult the experimental data in choosing those parameters.

The Belle collaboration observed the resonance-like structures Z_1 and Z_2 in the $\pi^+ \chi_{c1}$ invariant mass distribution near 4.1 GeV in the exclusive $\bar{B}^0 \rightarrow K^- \pi^+ \chi_{c1}$ decays [46]. The Breit-Wigner masses and the widths are about $M_1 = 4051 \pm 14_{-41}^{+20} \text{ MeV}$, $\Gamma_1 = 82_{-17-22}^{+21+47} \text{ MeV}$, $M_2 = 4248_{-29-35}^{+44+180} \text{ MeV}$ and $\Gamma_2 = 177_{-39-61}^{+54+316} \text{ MeV}$. If they are vector hidden charmed tetraquark states, the central value of the threshold parameter can be tentatively taken as $s_0 = (4.248 + 0.5)^2 \text{ GeV}^2 \approx 23 \text{ GeV}^2$, where we choose the separation between the ground states and first radial excited states to be 0.5 GeV .

The present experimental knowledge about the phenomenological hadronic spectral densities of the tetraquark states is rather vague, whether or not there exist tetraquark states is not confirmed with confidence, and no knowledge about the high resonances; we can borrow some ideas from the baryon spectra [57].

For the octet baryons with the quantum numbers $I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$, the mass of the proton (the ground state) is $M_p = 938 \text{ MeV}$, and the mass of the first radial excited state $N(1440)$ (the Roper resonance) is $M_{1440} = (1420-1470) \text{ MeV} \approx 1440 \text{ MeV}$ [57]. For the decuplet baryons with the quantum numbers $I(J^P) = \frac{3}{2}(\frac{3}{2}^+)$, the mass of the $\Delta(1232)$ (the ground state) is $M_{1232} = (1231-1233) \text{ MeV} \approx 1232 \text{ MeV}$, and the mass

of the first radial excited state $\Delta(1600)$ is $M_{1600} = (1550 - 1700) \text{ MeV} \approx 1600 \text{ MeV}$ [57]. The mass gap between the ground states and first radial excited states can be chosen as 0.5 GeV . In this article, the central value of the threshold parameter $s_0 = 23 \text{ GeV}^2$ makes sense.

However, the threshold parameter $s_0 = 23 \text{ GeV}^2$ cannot result in a reasonable Borel window, we have to postpone it tentatively to larger values. It is not an indication that non-existence of the vector hidden charmed tetraquark states below 4.7 GeV ; in other words, the QCD sum rules alone cannot indicate (non-) existence of the multiquark states strictly.

If the multiquark states exist indeed, we can release the criterion of pole dominance and take a more phenomenological analysis with the QCD sum rules. One may refuse the value extracted from continuum dominating QCD sum rules as quantitatively reliable if one insists on that the contribution from the pole term should be larger than (or about) 50% (for detailed discussions about this subject, one can consult Ref.[50]). In the present case, the numerical results indicate that the threshold parameter $s_0 > 30 \text{ GeV}^2$ can lead to possible Borel window, we take the value $s_0 = (32 \pm 1) \text{ GeV}^2$.

With the central values of the input parameters, the contribution of the condensate (of the largest dimension) $\langle \bar{q}q \rangle^2$ is less than 28% (32%) at the value $M^2 \geq 3.4 \text{ GeV}^2$, and the contribution decreases quickly to about 10% (11%) at the value $M^2 = 4.5 \text{ GeV}^2$ for the $C - C\gamma_\mu$ ($C\gamma_5 - C\gamma_\mu\gamma_5$) type interpolating current. In this article, the Borel parameter can be taken as $M^2 \geq 3.4 \text{ GeV}^2$, we expect the operator product expansion is convergent.

The contribution of the pole term is larger than 50% (49%) at the value $M^2 \leq 4.5 \text{ GeV}^2$, and the pole contribution is about (50 – 74)% ((49 – 71)%) at the value $M^2 = (3.4 - 4.5) \text{ GeV}^2$ for the $C - C\gamma_\mu$ ($C\gamma_5 - C\gamma_\mu\gamma_5$) type interpolating current, again we take the central values of the input parameters. If we take into account the uncertainty of the threshold parameter, $s_0 = (32 \pm 1) \text{ GeV}^2$, the pole contribution is about (48 – 77)% ((45 – 75)%). The Borel parameter can be taken as $M^2 = (3.4 - 4.5) \text{ GeV}^2$, where two criteria of the QCD sum rules are full filled [47, 48]. From the Figs.1-2, we can see that the sum rules are not stable enough below the value $M^2 = 3.8 \text{ GeV}^2$. In the article, the Borel parameter and the threshold parameter are taken as $M^2 = (3.8 - 4.5) \text{ GeV}^2$ and $s_0 = (32 \pm 1) \text{ GeV}^2$, respectively.

For the tetraquark states consist of light flavors, if the perturbative terms have the main contribution (in the conventional QCD sum rules, the perturbative terms are always have the main contribution), we can approximate the spectral density with the perturbative term (where the A are some numerical coefficients) [58],

$$B_M \Pi \sim A \int_0^\infty s^4 e^{-\frac{s}{M^2}} ds = AM^{10} \int_0^\infty t^4 e^{-t} dt, \quad (16)$$

take the pole dominance condition,

$$\frac{\int_0^{t_0} t^4 e^{-t} dt}{\int_0^\infty t^4 e^{-t} dt} \geq 50\%, \quad (17)$$

and obtain the approximated relation,

$$t_0 = \frac{s_0}{M^2} \geq 4.7. \quad (18)$$

The superpositions of different interpolating currents can only change the contributions from different terms in the operator product expansion, and improve convergence, they cannot change the leading behavior of the spectral density $\rho(s) \propto s^4$ of the perturbative term [58].

This relation is difficult to satisfy for the light flavor tetraquark states [50, 51, 52, 53, 54], however, it is not an indication that non-existence of the tetraquark states. The hidden charmed and bottomed tetraquark states, and open bottomed tetraquark states may satisfy the relation, as they always have larger Borel parameter M^2 and threshold parameter s_0 [20, 34, 59, 60]. Although the relation is derived for the light flavor quarks in the massless limit, the c and b are heavy quarks.

For examples, in Ref.[59], the authors take the $X(3872)$ as hidden charmed tetraquark state and calculate its mass with the QCD sum rules, the Borel parameter and threshold parameter are taken as $M^2 = (2.0-2.8) \text{ GeV}^2$ and $s_0 = (17-18) \text{ GeV}^2$; in Ref.[34], the authors take the $Z(4430)$ as hidden charmed molecular state and calculate its mass with $M^2 = (2.5-3.1) \text{ GeV}^2$ and $s_0 = (23-25) \text{ GeV}^2$. In those sum rules, the relation in Eq.(18) can be well satisfied.

In this article, $s_0/M^2 > 6.8$, the relation in Eq.(18) is certainly satisfied. The relation can serve as an additional constraint in choosing the Borel parameter and threshold parameter, it alone cannot lead to satisfactory results, as a number of values of the Borel parameter and threshold parameter satisfy the relation.

Taking into account all uncertainties of the input parameters, finally we obtain the values of the masses and pole residues of the Z , which are shown in Figs.(1-2),

$$\begin{aligned} M_Z &= (5.12 \pm 0.15) \text{ GeV}, \\ f_Z &= (1.31 \pm 0.26) \times 10^{-4} \text{ GeV}; \end{aligned} \quad (19)$$

$$\begin{aligned} M_Z &= (5.16 \pm 0.16) \text{ GeV}, \\ f_Z &= (1.25 \pm 0.25) \times 10^{-4} \text{ GeV}, \end{aligned} \quad (20)$$

for the $C - C\gamma_\mu$ type and $C\gamma_5 - C\gamma_\mu\gamma_5$ type interpolating currents respectively. In numerical calculations, we observe the uncertainties come from the parameter m_0^2 are very small, while the uncertainties come from the parameters m_c , $\langle \bar{q}q \rangle$ and s_0 are comparable with each other.

If we take larger uncertainty for the pole mass, $m_c = (1.3 \pm 0.2) \text{ GeV}$, the values of the mass change to $M_Z = (5.12 \pm 0.28) \text{ GeV}$ and $M_Z = (5.16 \pm 0.30) \text{ GeV}$ respectively. Furthermore, we vary the energy scale from $\mu^2 = m_c^2$ to $\mu^2 = 4.5 \text{ GeV}^2$, the central value of the mass M_Z changes slowly, less than 0.05 GeV .

The value $\sqrt{s_0} - M_Z \approx 0.5 \text{ GeV}$ happens to be the energy gap between the ground states and first radial excited states of the light baryons [57], the threshold parameter $s_0 = (32 \pm 1) \text{ GeV}^2$ makes sense.

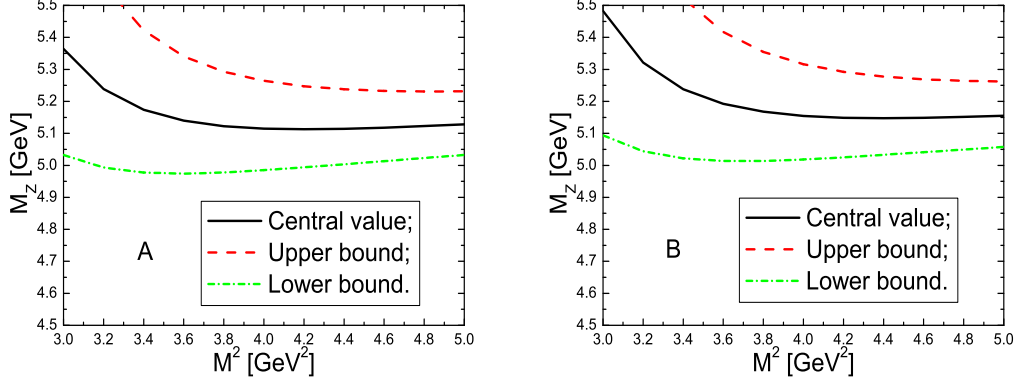


Figure 1: The mass M_Z with variation of the Borel parameter M^2 , A for the $C-C\gamma_\mu$ type current and B for $C\gamma_5 - C\gamma_\mu\gamma_5$ type current.

At the energy scale $\mu = 2 \text{ GeV}$, $\frac{\alpha_s}{\pi} \approx 0.09$ [57], if the perturbative $\mathcal{O}(\alpha_s)$ corrections to the perturbative term are companied with large numerical factors, $1 + \xi(s, m_c) \frac{\alpha_s}{\pi}$, for example, $\xi(s, m_c) > \frac{\pi}{\alpha_s} \approx 10$, the contributions may be large. We can make a crude estimation by multiplying the perturbative term with a numerical factor, say $1 + \xi(s, m_c) \frac{\alpha_s}{\pi} = 2$, the mass M_Z decreases slightly, about 0.1 GeV, the pole residue changes remarkably. The main contribution comes from the perturbative term, the large corrections in the numerator and denominator cancel out with each other (see Eq.(15)). In fact, the $\xi(s, m_c)$ are complicated functions of the energy s and the mass m_c , such a crude estimation may underestimate the $\mathcal{O}(\alpha_s)$ corrections, the uncertainties originate from the $\mathcal{O}(\alpha_s)$ corrections maybe larger.

The hidden charmed tetraquark state with the spin-parity 1^- lie in the region $(5.0 - 5.3) \text{ GeV}$, the mesons Z_1 , Z_2 or Z (about $(4.1 - 4.3) \text{ GeV}$) in the $\pi^+\chi_{c1}$ invariant mass distribution cannot be vector tetraquark states. The spins of the Z are not determined yet, they may be scalar hidden charmed states which may lie in the region $(4.1 - 4.3) \text{ GeV}$ [49], or more likely, they are hadro-charmonium resonances [14], P -wave $D^+\bar{D}^0$ molecular states, or $D_1^+\bar{D}^0 + D^+\bar{D}_1^0$ molecular states; more experimental data are still needed to identify them.

4 Conclusion

In this article, we assume that there exist hidden charmed tetraquark states with the spin-parity $J^P = 1^-$, and calculate their masses with the QCD sum rules. The numerical result indicates that the masses of the vector hidden charmed tetraquark states are about $M_Z = (5.12 \pm 0.15) \text{ GeV}$ or $M_Z = (5.16 \pm 0.16) \text{ GeV}$, which are inconsistent with the experimental data on the $\pi^+\chi_{c1}$ invariant mass distribution. The hidden charmed mesons Z_1 , Z_2 or Z may be scalar hidden charmed

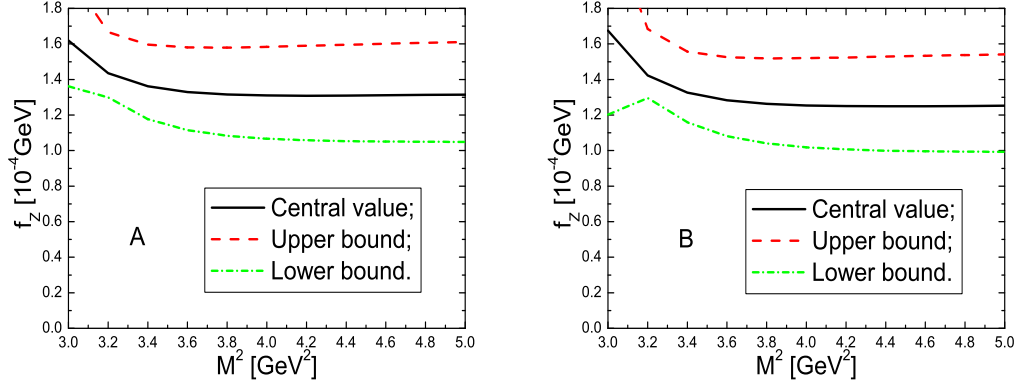


Figure 2: The pole residue f_Z with variation of the Borel parameter M^2 , A for the $C - C\gamma_\mu$ type current and B for $C\gamma_5 - C\gamma_\mu\gamma_5$ type current.

states, hadro-charmonium resonances or molecular states.

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